Profit Loadings in General Insurance Pricing

(A Critical Assessment of Approaches)

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Part I

① Introduction

The business of general insurance, like any other market based concern, is required to return adequate compensation on the capital, or surplus, supplied to it. Fundamental to fulfilment of this objective is the achievement of an appropriate price, or premium to cover the cost of providing the product (in the form of losses and expenses) and to provide a profit margin to service the capital required to conduct the business.

The cost of product provision is referred to as the pure premium (being the claims costs) plus expenses. The required premium is generally formulated as:

$$\text{Premium required} = \text{Pure Premium} + \text{Expenses} + \text{Profit Loading}$$

or

$$P = (1+\lambda)\, E[X]$$

where:

$E[X]$ is the expected cost of losses and expenses
λ is the profit loading
P is the required premium

In this paper we critically assess several methodologies for determination of an appropriate profit margin. We have examined some theoretical ideas proposed by actuaries and financial economists and discuss our view of the fundamental elements of each approach as well as practical considerations for implementation.

Methodologies considered include the surplus-return framework including the Insurance CAPM, ideas by Myers and Cohn, an options pricing based approach, utility theory and the proportional hazards transform.

The pricing of an insurance contract involves many factors including market or demand considerations, current and historical prices and forecast costs. In this paper we focus on one element of cost considerations, being the profit margin. It is important to recognise, however, that this is one element in the dynamic process of general insurance pricing.

② Surplus – Return Framework

The concept of a surplus-return framework is probably the most intuitively familiar methodology for determination of profit margins for most actuaries. Hart, Buchannan and Howe (1996) discuss a methodology based on surplus-return principles. The method relies on two key assumptions: a level of surplus that is allocated to a particular block of business, and a rate of return that needs to be earned on this surplus to meet shareholder expectations.

As a block of business is written it must be backed by assets of the insurer. The risk is funded partly through the premium paid and additionally from surplus. As the risk runs off this surplus is released. In a surplus-return framework, the profit margin is calculated as the difference between the present value of the surplus released discounted at the interest rate earned on the assets in which the surplus is invested (ie the locked in cost of capital) and the present value discounted at the required rate of return.

Thus, to determine the appropriate profit loading for a block of business using this framework we are required to:

- determine an appropriate level of surplus to be allocated to the business; and
- determine an appropriate return on that surplus.

2.1 Surplus Allocation

Many insurers take a simplistic approach and allocate surplus notionally between lines of business. Such an approach may be based on the written premium or outstanding claims or some combination of the two. However such simplistic approaches do not provide an adequate solution for the purposes of determining profit margins. They neither give any indication of the overall level of surplus required nor allow for differing relationships between risk and the proxy used for apportionment between lines of business. Most practitioners would agree that business which is subject to more uncertainty requires a higher level of surplus to support it (all other things being equal).

Some insurers may target a margin to hold above a statutory capital requirement in order to gauge a reasonable overall capital requirement for the business. Of course this still leaves the issue of appropriate apportionment of this capital by class of business.

More sophisticated methods of allocating capital based on the variability of the insurer's business have been advocated and we give a brief description of some of these below.

Process Risk and Parameter Risk

When considering the notion of risk in this context it is important to distinguish between process risk and parameter risk or equivalently, diversifiable and systematic risk. Process risk consists of risks such as the risk of loss on a single policy. Theoretically, where an insurer has a portfolio of a sufficiently large number of insurance policies, the amount of losses over the portfolio should become reasonably predictable and hence inherently less risky. The process risk attaching to a loss on any one policy can thus be "averaged out" or diversified away.

An insurer with a large portfolio of policies is still left with risks that will affect the level of losses overall (for example, superimposed inflation or legislative change). For a well diversified insurance company the risk that dominates the portfolio is the risk that the overall parameters associated with the book (for example, the estimate of the average loss amount) are not stable over time and hence subject to inherent uncertainty. This parameter risk cannot be diversified away.

The Standard Deviation Principle

A method commonly used to allocate capital between lines of business is the standard deviation principle. Under this method it is assumed that the riskiness of a block of business can be expressed as a multiple of the standard deviation of the present value of claims (and expenses) of the business at the earned interest rate. The standard deviation includes both process and parameter risk.

It is often thus assumed that the surplus required to support the writing of a block of business can be expressed as a multiple of the standard deviation. The stand deviation is only one of the possible risk proxies which could be used. It is possible to use a number of other proxies such as the probability of default, proportional hazards transform (discussed later) or some other risk measure. The principle in each case will be the same: the riskiness of the block of business is assumed to be some multiple of the risk measure chosen, and capital is allocated in proportion to this measure. We will discuss the method using standard deviation in more detail as it is widely cited as an approach.

The multiple of standard deviation used to describe riskiness is generally set to deliver some level of adequacy that the surplus apportioned (and overall) will prove adequate to meet the claims of the block of business (ie set to achieve an appropriate probability of ruin). Across a portfolio of many blocks of business the standard deviation of the total claims and expenses will not be equal to the sum of the parts. Therefore the multiple of standard deviation for each block of business must be scaled down to maintain an appropriate level of adequacy for the insurer as a whole or to allocate a given surplus between blocks of business. More formally, surplus allocation is required to account for the covariability between blocks of business.

Methods Based on Recognition of Covariability

The Method of Marginal Surplus

Under the method of marginal surplus, when an insurer adds a block of business to an existing portfolio, additional surplus is required such that the (acceptable) probability of ruin of the insurer is maintained. Under the formulation illustrated by Heckman (1992), if we consider the surplus of the insurer to be a random variable, then the initial surplus (S) held by the insurer is expressed as a multiple of the standard deviation of the surplus denoted:

$$S = Z.\sigma_s$$

where:

S = the initial surplus
σ_s = the standard deviation of the surplus
Z = the multiple of standard deviation of surplus giving the required probability of ruin over a suitably chosen time interval

As a block of business is added, the standard deviation of the insurer's surplus becomes, according to the standard statistical result for the sum of two random variables:

$$\sigma_s^1 = \sqrt{\sigma_s^2 + \sigma_i^2 + 2\rho\sigma_s\sigma_i}$$

where:

σ_s^1 = the standard deviation of the insurer's surplus with the additional block of business added
σ_i = the standard deviation of the present value of the additional block of business
ρ = the correlation coefficient between the present value of the additional block of business and the present value of the existing book

The required additional surplus is:

$$
\begin{aligned}
\Delta S &= Z\left(\sigma_s^1 - \sigma_s\right) \\
&= Z\frac{2\rho\sigma_s\sigma_i + \sigma_i^2}{\sigma_s^1 + \sigma_s}
\end{aligned}
$$

Note that this approach assumes that the factor Z chosen remains appropriate after the addition of this block of business. This requires the relationship between the cumulative percentage of the surplus distribution (which is captured by the product of Z and the standard deviation) and the standard deviation of the surplus distribution remains unchanged. Where the additional block of business is a small proportion of the existing portfolio this may be considered an appropriate assumption. The required change in surplus can be written as:

$$\Delta S \approx Z\rho\sigma_i$$

In this case the additional surplus is a product of the standard deviation of the additional risk, the correlation with the existing portfolio and the multiple Z. The major implication here is that if the risk is sufficiently small, it will only add to the insurer's surplus requirement to the extent that it is correlated with the existing book of business.

Gogol's Approach

Gogol (1996) formulates a method to deal with allocation of surplus to underwriting and outstanding claims by line of business. The formula is essentially the same as the method of marginal surplus above, however there is no explicit consideration of an appropriate probability of adequacy ie the methodology describes allocation of existing surplus.

Gogol states that surplus variability is caused by the following "categories" of risk:

- Underwriting;
- Outstanding claims and offsetting assets; and
- Other assets.

Surplus is allocated to all sources of variability in proportion to:

$$\frac{\text{Cov (Total surplus, Effect of category on total surplus)}}{\text{Var (Total surplus)}}$$

Here the covariance of a sum of categories equals the sum of covariances so that if the sources of variability are referred to as categories 1 through n we have:

$$\sum_{i=1}^{n} \text{Cov (Surplus, Effect of category i on surplus)}$$

$$= \text{Cov (Surplus, Effect of (}\sum_{i=1}^{n}\text{ Category i) on surplus)}$$

$$= \text{Cov (Surplus, Surplus)}$$

$$= \text{Var (Surplus)}$$

So the sum of the amount of surplus allocated to each category equals the total amount of surplus.

Considering the effect on the variance of surplus of category i and denoting the covariability of category i with all other sources of variability (i.e. the surplus excluding i) as Cov(i,s\i) then:

$$\begin{aligned} \text{Cov (i,s)} &= \text{Cov (i,i)} + \text{Cov (i,s\\i)} \\ &= \sigma_i^2 + \sigma_i \sigma_{s\backslash i} \rho_{i,s\backslash i} \end{aligned}$$

where:

s = surplus
σ_i = standard deviation of category i
σ_s = standard deviation of surplus
$\rho_{i,s\backslash i}$ = correlation between category i and surplus

Then, if category i is sufficiently small compared to overall surplus

$$\text{Cov(i,s)} \approx \sigma_i \sigma_{s\backslash i} \rho_{i,s\backslash i}$$

which can be seen to be similar expression as for the method of marginal surplus.

Gogol comments on the practical difficulties involved in estimation of covariances. He suggests a method whereby surplus is first allocated between outstanding claims and underwriting. Surplus is then allocated between underwriting of short-tail and long-tail lines and so on until surplus is allocated at the appropriate level. For large classes of business, historic data may be adequate to estimate covariances however, for smaller classes and in practice overall, a significant amount of judgement will be required.

Simulation Approaches

The implementation of any methodology used for developing profit margins or allocating surplus by class of business will require a numerical solution to the functional form of the methodology considered. This solution can be developed either analytically, or using numerical methods such as some form of asset-liability modelling (ALM).

The main advantage of using an ALM approach is that it avoids the restrictions of needing to have analytically "nice" functions. However ALM approaches have their own set of problems. For example, a downside of not requiring analytically "nice" functions is that the resulting solution is not differentiable, making it harder to determine the marginal contribution of certain factors. In particular, when using ALM approaches for surplus allocation, a suitable numerical method for determining the marginal contribution of each class of business is required.

The data requirements for an ALM approach are quite different to the data requirements if an analytical solution is used. Successful use of stochastic techniques of this nature requires the development of empirical input distributions (or determine the appropriate parameters for statistical input distributions) for a host of quantities from claim sizes and frequencies to expense assumptions and investment return distributions.

One resultant draw back is the time-intensive nature of the ALM exercise. From a practical perspective, it is likely that the ALM approach will prove too lengthy an approach if it is done merely for the purposes of setting rates.

However, many companies do asset-liability modelling for other purposes, such as determining capital requirements or for capital allocation. As such, it is possible that the pricing actuary could use such results. It is important that the pricing actuary understands any limitations of the modelling and the model assumptions.

Conclusion

It is important that the allocation of surplus appropriately captures the variability of the risk. The contribution to variability of each element is ameliorated as all risks are not perfectly correlated and this must be recognised by the allocation process. The results of the process will only be as appropriate as the assumptions that are used.

2.2 Determination of Target Return on Surplus

Once a suitable surplus allocation for a block of business has been determined attention is focussed on the required return on that surplus in order to derive an appropriate profit margin.

In many practical contexts, the pricing actuary is required to determine a profit margin which will meet the target rate of return "set by management". This rate is presumably aligned to the expectations of shareholders in the case of a corporate. It must be such that the shareholders are adequately compensated for the risk borne by investment in the insurance company.

Whilst ultimately the target rate of return is a management decision, the pricing actuary should be in an informed position to advise management about the appropriateness of the target rate. This should take into account (amongst other factors) the riskiness of the block of business. One solution that is commonly used is the Capital Asset Pricing Model, which is discussed in the next section.

③ CAPM as a Solution for Return

3.1 Introduction

A widely used approach to address assessment of return expectations is the Capital Asset Pricing Model (or CAPM). CAPM is largely based on the concept of diversification. It assumes that risk consists of two components: diversifiable or security specific risk and non-diversifiable or systemic risk.

Investment in the securities market involves risks associated with an individual security that are security specific and risks that affect the market overall, eg inflation. The CAPM argument is that such systemic risk, which cannot be eliminated by diversification, is the only risk that should be rewarded by higher returns.

Systemic risk is priced through the beta of the security. Beta measures the component of the overall variability that cannot be diversified away by holding other assets.

Based on this, CAPM postulates that the expected rate of return for a particular security is given by:

$$E[R]= R_f + \beta * (E[R_m] - R_f)$$

where

R = portfolio rate of return
R_f = the rate of return on a risk-free asset
R_m = the rate of return on the market portfolio
β = the beta of the portfolio
= Covariance (R,R_m)/Variance (R_m)
E[] is the expectation operator

Central to CAPM is the concept of the market portfolio. The market portfolio is a theoretical construct consisting of all assets present in the market. It represents a fully diversified portfolio subject only to systemic risk. The difference between the expected return on this market portfolio and the risk-free rate represents the extra reward that is obtained by investing in one unit of non-diversifiable risk. The return for a particular portfolio should exceed the risk free rate only to the extent that it contributes to the systemic risk. This is measured by the beta of the portfolio as discussed.

3.2 The Insurance CAPM

CAPM has formed the basis for a number of models used to determine profit margins in insurance. One of the first of these was that which later became known as the Insurance CAPM. It was developed by William Fairley in the 1960s. The model played a role in determination of appropriate profit margins used in rate filings in Massachusetts, USA.

The Fairley model uses CAPM to derive risk adjusted rates of return that financial markets require of insurers. This is then converted to a profit margin by consideration of the expected term of liabilities through the ratio of premium to surplus in the manner illustrated below.

According to the Fairley model, the profit margin required is given by the expression:

$$r = -k\left(r_f + \beta_L (r_m - r_f)\right)$$

where:

r = profit margin required (per dollar of premium)
k = funds generating coefficient
β_L = beta of liabilities
r_m and r_f defined as per the basic CAPM formula outlined above

Note that in this simplified version of the Fairley model, the impact of taxes is ignored.

The funds generating coefficient can be likened to the ratio of technical reserves to premium. As the typical cash flow pattern in insurance consists of the early receipt of premiums followed by subsequent payment of claims and expenses, the insured is effectively making a “loan” of its cash flow to the insurer. The implication of the Fairley model is that a return at the risk free rate for the duration of the liabilities is an offset to the profit margin required by the insurer.

The beta of liabilities is analogous to the individual security beta used in the basic CAPM formula measured as:

$$\beta_L = \mathrm{cov}(r_L, r_m) / \mathrm{var}(r_m)$$

where

r_L = the underwriting return

It can be seen from the above expression for the profit margin that if the beta of the liabilities is negative then the contribution of $-k\beta_L(r_m - r_f)$ will be to increase the required profit margin. This represents the contribution to the insurer's profit margin from the systemic risks of underwriting. This is the only risk rewarded in the Fairley model. Fairley is also quick to point out that the profit margin will be greater the longer the term of the cash flows (ie the larger is k) and that this accords intuitively with the industry's view of the world.

Importantly the model assumes that the insurer is able to construct a "minimum risk portfolio" with the assets backing insurance liabilities (or technical reserves). The minimum risk portfolio is a portfolio consisting of some proportion of assets held in the market portfolio and some proportion held in risk free securities. The proportions are determined by the beta of the liabilities. So for example, a liability beta of 0.5 would imply a minimum risk portfolio consisting of 50% risk free assets and 50% of assets invested in the market portfolio. Indeed, a liability beta of –0.5 would imply a minimum risk portfolio consisting of a short sale of 50% of the value of the technical provisions in the market portfolio with 150% invested in the risk free asset.

It is also important to note that with the Insurance CAPM, the insurance profit is expected to be zero. The risk premium along with the expected return on technical provisions is expected to meet claim and expense payments exactly.

Fairley empirically measured the beta of liabilities to be -0.21. The empirical measurement of the beta of liabilities is fraught with difficulties (see the discussion below). One of the major issues confronting the Fairley approach is that traded insurance companies, the data of which was used to measure the beta of liabilities are multiline. It had been shown that use of accounting data by class of business would bias the beta estimate and Fairley was limited to a single beta to apply to all classes of business.

3.3 An Alternative "Insurance CAPM"

An alternative form of the "Insurance CAPM" has been proposed by Feldblum (1990). Feldblum argues that it is incorrect to measure the systemic risk of underwriting through the mechanism of the underwriting beta (or liability beta) as proposed by Fairley. Feldblum argues that the underwriting beta measured as proposed by Fairley measures the systemic risk faced by shareholders in insurance firms rather than the systemic risk faced by the insurer. He contends that an insurer does not have the option that a shareholder has of investing its assets in securities other than the risk free asset and using it to support the insurance written. The argument is that investment in other securities by the insurer would subject the investor to double taxation (ie taxation paid by the insurer on its investment earnings and then tax paid by the investor on dividends and capital gains). Shareholders in insurance companies, he suggests, would rather invest their equity directly and not subject it to double taxation. This argument can be seen to break down somewhat in the presence of dividend imputation.

Feldlum's model replaced the liability beta as utilised by Fairley with a coefficient for systemic risk measured against a diversified insurance portfolio:

$$r = r_f + \beta_F (r_p - r_f)$$

where:

r = Feldblum's return on an insurer's equity
r_f = the risk free rate
β_F = "Feldblum's beta"
r_p = the return on a "fully" diversified insurance portfolio

The return as expressed is proposed as the appropriate return on an insurer's capital. It is converted to a profit margin by application of an appropriate premium to surplus ratio.

Feldblum's beta is estimated by regression of individual class of business profit margins (ie 1 less the combined operating ratio, which is the sum of the loss ratio and expense ratio) against industry returns. Feldblum found higher betas generally for long-tail lines of insurance versus short-tail lines. (Notable exceptions were Worker's Compensation and Auto Liability where, he argued, market regulation or other mechanisms kept profits low and stable). The Feldblum model produced higher required returns than did the Fairley model (this could in some part be due to Feldblum's proposed rate of return on the fully diversified portfolio).

3.4 Feldblum vs Fairley

It is interesting to compare the Fairley and Feldblum models although it is not simple to make a direct mathematical comparison. In substance, the major difference in the methods is the essence of the risk being rewarded. It is implicitly assumed in the Fairley model that an insurer has the same opportunity to diversify its asset holdings backing the technical liabilities as any investor in the market and, as such, the only risk rewarded is the liability risk which cannot be diversified away through an optimal asset holding.

Feldblum asserts that an insurer does not have perfect freedom in its asset allocation and hence could not achieve the optimal portfolio assumed by Fairley. For Feldblum, the risk rewarded is that which remains after formation of a fully diversified insurance portfolio ie Feldblum assumes that all assets backing technical provisions must be invested so as to directly support the insurance writings and hence that the insurer has no freedom in investment of these assets. Intuitively this would mean that the return under the Feldblum model should exceed that under the Fairley model.

It is generally considered best practice for an insurer to invest the assets backing the technical liabilities in a risk-free duration matched portfolio. This is due to the impact of statutory requirements and the “short-termism” in investment strategy resulting from annual and semi-annual reporting periods. In view of this it would be a courageous step for an insurer to invest the assets backing the technical provisions in the “minimum risk portfolio” as previously defined and as proposed by Fairley.

Under both the Fairley and Feldblum formulations it is assumed that the insurer is free to invest the surplus assets in an unrestricted manner. Of course, in practice there will be restrictions on investment of these funds. The insurer will be required to meet statutory solvency obligations and this will generally necessitate restrictions in the manner in which surplus assets are invested.

In practice the restrictions on the investment of surplus assets impose an additional cost on the shareholders which also must be compensated via the profit margin. The investment decision in respect of surplus assets should be considered within a risk/return framework. This framework should appropriately recognise the allocation of the assets backing the technical liabilities, the insurance liabilities themselves and the restrictions on the investment opportunities imposed on the insurer in relation to its surplus funds.

As neither the Fairley nor Feldblum formulations recognise this additional cost it appears that both may provide inadequate compensation to shareholders of insurance companies.

3.5 Criticisms of CAPM

Along with the apparent elegance of CAPM come some fairly restrictive assumptions, namely that:

- markets are efficient;
- all investors have identical expectations of returns and equal information;
- there are no taxes or transaction costs;
- it is possible to lend and borrow at the risk-free rate;
- all investors have the same time horizon;
- it is not possible for an individual investor to have an impact on market prices; and

- it is possible to trade in assets in infinitesimally small quantities.

It is plausible that we could challenge each one of these assumptions in a practical context. A number of authors have extended CAPM to cope with multiple time periods and taxation. However, even these extended versions of CAPM are still reliant on many of the assumptions cited above.

One of the major issues with CAPM is the problematic nature of beta estimation. The issues apply to both the Fairley and Feldblum formulations of beta. In his 1994 paper, Thomas Kozik discuses the large inconsistencies in estimates of betas in published literature. He states quite emphatically that "underwriting betas are not observable".

Kozik explains that because insurers undertake investment and underwriting operations the estimation of underwriting betas is complicated. Two methods are generally used in the literature and the beta estimates so obtained are referred to by Kozik as:

- accounting betas: historic accounting underwriting returns regressed on a corresponding history of the returns on a market index;
- inferred betas: assuming that an insurer's equity beta is a linear combination of its asset beta and underwriting (or liability beta) betas, the underwriting betas is inferred from the insurer's asset and equity beta.

From Kozik's evidence, the large inconsistencies in empirical measurement of underwriting betas are caused by one or a number of the following factors:

- insurance companies used in the study;
- the time period used for estimation (both the time over which the study was conducted and the frequency of the period used for the return measurement);
- the choice of market portfolio; and
- the methodology used for estimation.

Such major inconsistencies in the measurement of the primary parameter in the insurance CAPM are of concern.

Kozik also points out that CAPM is concerned with return expectations from which historical realised returns may and most likely will diverge. It is thus questionable whether estimation of an ex-post (historically measured) beta is even relevant to the CAPM in its ex-ante (expected) form.

In addition to the problems with beta estimation the Insurance CAPM is also criticised for its failure to recognise default risk. The crudeness of the factor k to represent the runoff process is also cited as a limitation. The timing of cash flows is the critical essence of the modification proposed in the Myers-Cohn model and this is discussed in the next section.

3.6 Conclusions

The Fairley model critically depends on the flexibility of an insurer to invest the assets backing the technical provisions in such a manner as to construct the minimum risk portfolio. We have questioned the assumption citing generally accepted industry practice, which is driven by statutory and short-term reporting requirements. These indicate that an insurer should minimise risk by investing the assets backing the technical provisions in a risk free portfolio of duration matched securities.

Feldblum recognises this restriction and hence postulates a more acceptable return framework.

Both methods fail to recognise the additional cost imposed on shareholders through restriction on the investment strategy with respect to surplus assets.

Additionally, the measurement of the parameter fundamental to either the Fairley or Feldblum formulation is fraught with difficulty. This raises additional concern over the use of CAPM based methodology.

④ The Myers-Cohn Model

4.1 Introduction

Stewart Myers and Richard Cohn developed a model for determination of profit margins based on discounted cash flow (DCF) concepts in 1981. This model was developed in response to the Fairley model and attempted to address some of the weaknesses inherent in the Fairley formulation. Specifically, it refines the treatment of the claim payments and other outgoings by explicitly allowing for the timing of cash flows.

The formulation of the Myers-Cohn model begins with the premise of an "idealised balance sheet". The assets are equal to the market value of the financial assets of the insurance company, and are increased by premium revenue. The liabilities consist of:

- policy reserves (being the present value of expected losses and expenses)
- the present value of the liability for income tax and
- the equity value.

Myers and Cohn state that the premium should be "fair" to both the shareholders and the policyholders of an insurance company. Essentially, policyholders pay shareholders an amount commensurate with the expected value of losses, expenses and taxes and additionally an amount to compensate for the transfer of the risk. This premium is fair in that there is no net transfer of wealth between the policyholders and the shareholders. The equity value is represented by the financial assets less the present value of losses and expenses and is hence equal to the equity investment by the shareholders.

4.2 Formulation of the Model

The Myers-Cohn model gives the fair premium (P) by the following formula:

$$PV(P) = PV_{rL}(L) + PV_{rf,\,rL}(UWPT) + PV_{rf}(IBT)$$

where

$PV_{rL}(L)$ = Present value of the liabilities and expenses, discounted at rate r_L

$PV_{rf,\,rL}(UWPT)$ = Present value of the tax on underwriting profit. The underwriting profit will depend on the level of premium income, losses and expenses. The premiums are discounted at rate r_f and the losses and expenses are discounted at rate r_L

$PV_{rf}(IBT)$ = Present value of the tax on the investment income, discounted at rate r_f

Risk-free cash flows (such as the premium income) are discounted at the risk-free rate, r_f, whereas cash flows that are subject to some degree of risk are discounted at a risk-adjusted rate r_L. In the formulation given above, the liabilities or claims and expenses are subject to variability, and are hence discounted at a risk adjusted rate. The rest of the cash flows, however, are assumed to be risk-free. Premiums are received in advance in the general case and are hence known with certainty. The tax on investment income is discounted at the risk free rate. The appropriateness of the use of the risk free rate was shown not to be dependent on the assets being invested in risk free securities.

The model critically depends on the rate used to discount the risky cash flows. While it is theoretically possible to use a discount rate developed in one of a number of appropriate ways, including a benchmark or "set by management" approach, it is often believed that the Myers-Cohn model is synonymous with the CAPM framework. This is not the case. In their original formulation Myers and Cohn explicitly state that the discount rate does not rely on the Insurance CAPM.

The key to the Myers-Cohn framework is the use of different (risk adjusted) discount rates to value different sets of cash flows. The appropriate risk adjusted discount rate is thus a key assumption. At present no real alternatives to the Insurance CAPM have been proposed for estimation of this rate. Myers and Cohn do not propose alternative ways of developing the discount rate, despite stating that the methodology is not dependent upon the CAPM framework. From a practical perspective, the lack of alternative methodologies means that in practice, it has proved difficult to divorce the Myers-Cohn model from the Insurance CAPM.

Myers and Cohn develop the appropriate risk-adjusted rate using the Insurance CAPM prescribed by Fairley as described in the previous section. Thus:

$$r_L \quad = \quad r_f + \beta_L (r_m - r_f)$$

The approach adopted by Myers-Cohn in relation to the discount rate adopted is hence subject to the inherent weaknesses that underlie the Insurance CAPM, which were discussed at length in the previous section.

4.3 Insurance Capital and the Myers-Cohn Model

The Myers-Cohn model is relatively insensitive to the level of capital supporting the business. The only term in the formulation impacted by the level of capital held is the present value of the taxes on investment income.

According to the Myers-Cohn formulation this, combined with the net income arising out of the underwriting process, represents the opportunity cost to the investor of not investing directly and is hence the only additional reward due to the investor.

Note that the importance of this term would decrease in the presence of dividend imputation in which case the investor receives a franking credit to offset the corporate tax paid on the investment income.

In the Myers-Cohn formulation, if the capital held to support a block of business was doubled, the only increase in "fair" premium would be an allowance for the extra taxes that would have to be paid on the investment income on this capital.

Most practitioners would argue that a doubling in the capital required to support a particular block of business should result in a significant increase in profits to adequately compensate shareholders. Essentially, the Myers-Cohn model assumes that the investment income generated on the surplus will be sufficient to compensate shareholders for this investment (apart from some slight taxation effects). This may encourage investment in risky assets such as equities to ensure that the investor receives an adequate reward. If there are any restrictions on the asset allocation of the insurer, for example due to regulatory constraints, then there may be additional opportunity costs to the investor, which have not been allowed for in the Myers-Cohn model.

4.4 Myers-Cohn in Summary

The DCF framework is one on which actuaries are quite well versed and hence, the Myers-Cohn framework is intuitively appealing.

The methodology has a number of advantages:

- provided an appropriate discount rate, the only requirement is to calculate the present value of a set of cash flows. This is a reasonably simple process;
- it incorporates loss payment patterns in a more explicit manner than earlier models.

However, the Myers-Cohn model has two fundamental weaknesses:

1. The profit margin determined using the Myers-Cohn model is inseparable from the risk-adjusted discount rate used. One approach to determining this discount rate depends on the Insurance CAPM, which is subject to a number of difficulties that have been discussed previously. Of course, the practitioner could choose not to use the Insurance CAPM to

determine the risk-adjusted discount rate. However, at present no real alternatives have been proposed.

2. The Myers-Cohn model is relative insensitive to the amount of capital supporting the business. If we assume that the rate of return on surplus is adequate then this insensitivity to the actual capital held is not unreasonable, however, if this is not the case then the return implied by the Myers-Cohn formulation will prove inadequate.

⑤ Application of Option Pricing

5.1 Introduction

An option is a derivative security. It derives its value from the price of the underlying asset. Typical options are based on stocks, bonds and indices. The owner of the option has the right but not the obligation to trade the underlying security at or within a future time at a specified price.

A call option gives the option owner the right to buy the underlying asset at a specified (or exercise) price. A put option gives the option owner the right to sell the underlying security. Types of options include "American Options" or "European Options". American options may be exercised any time up until their expiration date. European options may be exercised only on the expiry date.

The pay-off (or the amount received at expiry) for a European Call option may be expressed as:

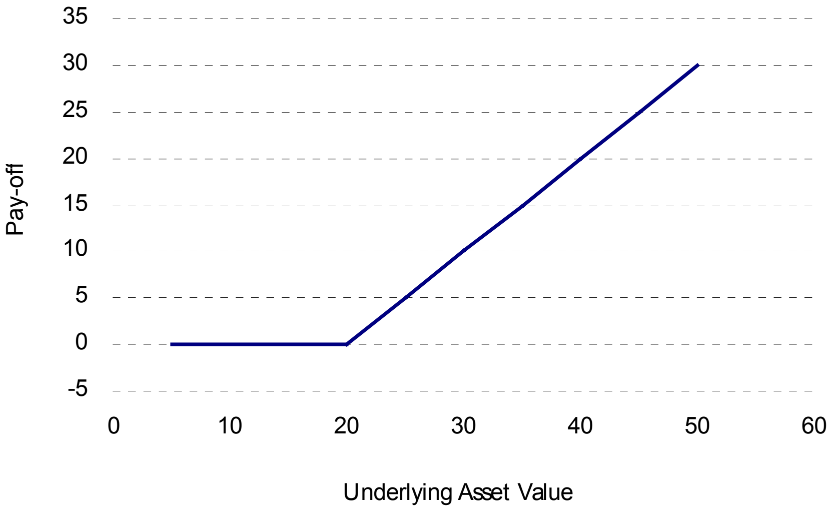
$$P = \max[S-K,0]$$

Where

P = Value of Call option at expiry
S = Price of the underlying security when the option is exercised
K = Exercise price of the option

Figure 5.1 shows the pay-off to the owner of a call option on an underlying asset with exercise price 20, for varying asset values:

Figure 5.1 – Payoff from a Simple European Call Option



Similarly, the pay-off for a European Put option may be expressed as:

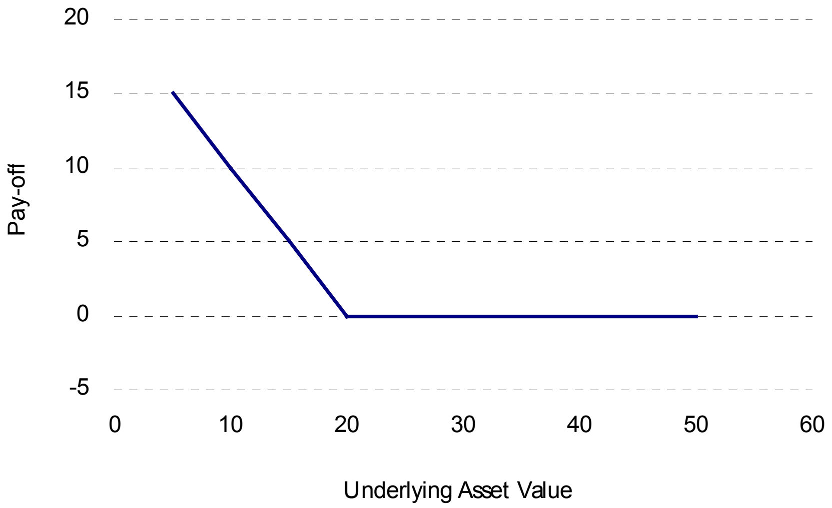
P = Max[K-S,0]

Where

P = Value of Put option at expiry
S = Price of the underlying security when the option is exercised
K = Exercise price of the option

Figure 5.2 below shows the pay-off to the owner of the put option on an underlying asset with exercise price 20, for varying asset values:

Figure 5.2 – Payoff from a Simple European Put Option



A number of methods have been proposed to calculate the value of the option before expiry. Arguably the model devised in 1973 by Fisher Black and Myron Scholes has become one of the most popular models.

The Black-Scholes model considers five factors that affect option prices:

- the current value of the underlying asset
- the exercise price
- the time to expiry
- the volatility of the price of the underlying asset
- the risk-free rate of interest.

The problem of pricing the option then becomes one of determining sensible estimates for each of these parameters.

5.2 Options and Insurance

In the case of property insurance, an insurance contract depends on the value of other, underlying assets. The policyholder is insuring the value of this asset against the occurrence of specified events which lead to the diminution of this value. A liability insurance cover could be viewed similarly where a "negative asset" could be represented by the loss sustained by the occurrence of an event. Some practitioners have suggested that options-based pricing techniques may be appropriate for pricing insurance contracts. To make the following discussion simpler we focus on a contract of property insurance however we intend the argument to extend to liability cover.

In this framework, it is possible to think of an individual buying an insurance policy as buying a put option. When a specified loss occurs, the asset value drops and the insured receives a positive payoff. In the case of a total loss, the insured has the right to lodge a claim, or equivalently to sell the underlying asset (e.g. their house) to the insurance company for an amount equal to the sum insured less the excess. If the loss is partial, the insured will receive the difference in value between the sum insured less the excess and the lower value of the asset. This could be seen as equivalent to having a put option on the asset with its exercise price equal to the sum insured less the excess. It has been proposed that determining a premium for this put option can be viewed as determining the value of this equivalent put option.

Each of the parameters in the Black-Scholes model can be seen to have an equivalent in the context of insurance pricing:

- the current value of the underlying asset is the value of the insured asset (e.g. the house);
- the exercise price is the sum insured less the excess;
- the time to expiry is the policy term;
- the volatility of the price of the underlying asset is derived from the value of the insured asset; and
- the risk-free rate of interest is the rate of interest on a government bond of equivalent term to the insurance liabilities.

Some researchers have suggested that the Black-Scholes model can be used for pricing insurance contracts.

Strictly, the insurance policy is analogous to an American option (i.e. it is possible to exercise it at any time up to expiry) and unfortunately there are no simple methods of valuing American options. Indeed it is possible that the insured may incur several claims during the policy term further complicating the calculation. It may be possible to rely on simulation techniques to value these “options” or to make simplifying assumptions such as any loss is not paid until policy expiry, in which case it may be possible to approximate the calculation using the Black-Scholes model.

It is important to ensure that the volatility of the underlying asset refers only to the gyrations in value caused by insured events and not to variations in the market price of the asset that are unrelated to insured events. For example, in the case of household insurance it is important to distinguish between a fall in value due to insured events (eg a storm) and falls in value due to a decrease in housing prices.

Several researchers have suggested option-pricing based methods that consider the insurer’s book of business in aggregate. These methods consider the limited liability of shareholders and focus on the possibility of default. Thus, they consider the problem in terms of the shareholders having an option to pay less than the full value of the losses. Shareholders implicitly purchase this option from policyholders through lower premiums. The final formulation is that the value of the shareholders equity is equal to the difference between two call options (valued using the Black-Scholes Model), both dependent upon the premium. This equation can then be solved to determine the profit margin. See D’Arcy and Dyer (1997) for a more detailed exposition of this approach.

5.3 An Option Pricing Approach in Practice

On the surface, an option-pricing based methodology may appear to have certain appealing attributes when compared to other pricing methodologies:

- Option-pricing approaches do not require a target rate of return or estimates of beta
- Some of the option-pricing models proposed allow for possible insolvency and the limited liability of shareholders.

On the other hand, there are a number of problems with applying these methodologies. Many of the problems discussed are specific to the use of the Black-Scholes model to valuing options. Because, as already discussed, this model is ubiquitous any approach to valuing insurance contracts that relies on option-based methodologies will almost invariably rely on this model.

The disadvantages of option-pricing based methodologies are:

- Time invariant volatility and risk-free rate assumptions

  The Black-Scholes methodology assumes that the volatility of the underlying asset and the risk-free rate of interest remain constant over the time period considered. In traditional derivatives markets where the Black-Scholes model is used, the time period until expiry is usually of sufficiently short duration for this simplification to not be too extreme. However, when considering insurance contracts the time period in question is typically too long for this to represent a valid assumption.

- Single term to expiry

  Options pricing models assume a single time to expiry. It is a considerable simplification to assume that liabilities are paid at a single time in the future

- Other assumptions that are doubtful in practice

  The model relies on a number of other assumptions that are not necessarily borne out in practice, namely, it is assumed that

  - it is possible to borrow and lend at the risk-free rate
  - it is possible to short-sell the underlying asset
  - the underlying asset is infinitely divisible
  - the underlying asset is continuously traded
  - there are no taxes or transaction costs
  - individual trades do not affect the price of the underlying asset
  - the market is efficient (i.e. all participants in the market have complete information and assets always trade at their fair value)

- the value of the underlying asset follows a lognormal distribution.

A number of the assumptions listed above are necessary for the continuous hedging argument that is fundamental to the Black-Scholes model. Yet these assumptions break down in an insurance context.

The continuous hedging argument states that it is possible to exactly replicate the payoffs of the option by holding a suitable combination of cash and the underlying asset. The theory is that by constantly rebalancing the portfolio one is able to always hold the appropriate combination to replicate the payoff.

Also, due to the constant rebalancing, the strategy is assumed to be self-financing (i.e. there are no cash flows in or out required and the strategy is essentially risk-free). As it is possible to exactly replicate the payoffs using a combination of assets that is known at the outset the price of the option must be equivalent to the value of these assets.

Underlying this process is the assumption that the price of the risky asset is not subject to sudden jumps in its value. Here too this is not the case for insurance assets. For example, the change in value of a car involved in an accident is not a continuous process during which the asset can be traded to reduce the holding in the vehicle. The violation of this fundamental concept underpinning the Black-Scholes methodology means it is inappropriate to apply in an insurance context.

In the context of the insurance portfolio as a whole one might assume that price discontinuities of the type experienced in an individual contract might average out. However the model still relies on the continuous trading of the assets and liabilities of the insurer, including the ability to short-sell assets and liabilities as required. This is not the case in practice, with no active secondary market in insurance liabilities.

5.4 Concluding Remarks

Although the concept of using option pricing based techniques in insurance pricing may be intuitively appealing, these should be approached with caution. Traditional approaches to valuing options using the Black-Scholes methodology are not necessarily appropriate in an insurance context.

Option pricing based techniques are unlikely to provide a feasible method for determining the appropriate profit margins either due to theoretical shortcomings or practical difficulties in the modelling.

⑥ Utility Theory

In this section we consider some insight from utility theory on the question of setting profit margins. We give a brief refresher on the concept of utility theory, discuss the structure and parameterisation of an appropriate utility function and consider the appealing aspects of the approach as well as some criticisms that have arisen.

6.1 What is Utility Theory

Basic concepts concerning Utility Theory (or Subject Value Theory) first surfaced in economic literature in around the 1870s in works published virtually simultaneously by William Stanley Jevons (UK), Karl Menger (Austria) and Lèon Walras (France). In microeconomics utility commonly refers to the satisfaction or pleasure that a consumer receives from the consumption of goods or services. In the present context, the utility of an insurer relates to the value it derives from a given level of wealth or surplus.

In applying utility theory to the determination of profit margins the problem can be stated thus: An insurer should write a policy if the expected utility of the insurer's surplus is not diminished by writing the contract. An insurer's surplus is impacted by the writing of a risky insurance contract given the uncertain outcome of the insurance process.

The manner in which an insurer's expected utility varies with varying levels of surplus (as the outcome of engaging in a risky transaction) conveys a message concerning the insurer's risk preference. Risk preference could be described as risk averse (risk loving) in which case the increase (decrease) in the utility derived from an added (subtracted) unit of surplus is less than the decrease (increase) in utility derived from a subtracted (added) unit of surplus. Between these descriptions of risk preference lies risk neutrality.

Intuitively we would describe an insurer as risk averse. As such, an insurer will engage in a risky transaction to the extent that its expected utility is not decreased. The addition of a risk margin to the pure premium on the expected loss of an insurance contract is required to induce a risk averse insurer to engage in the transaction.

6.2 The Utility Function

The utility function is the mathematical description of an insurer's attitude toward risk. An appropriate utility function requires several properties:

- It must be monotonically increasing – an insurer would always prefer increasing wealth (surplus);
- It should be differentiable – this will allow us to describe the relationship between surplus and marginal utility;
- It should show decreasing marginal utility to accord with our intuitive view of the world.

The function would also have intuitive appeal if risk aversion decreases as surplus increases. In this sense, an insurer would be more inclined to engage in a given risky transaction as its surplus increased.

A popular functional form satisfying most of the above criterion is the exponential utility function (note that the function is flexible enough to convey risk neutral or risk loving preferences as well as risk aversion). The exponential utility function however does not exhibit "wealth effects": the initial surplus of the insurer is irrelevant to the required risk margin. This effect can however be included in this formulation by allowing the parameter in the formulation below to vary with surplus

Halliwell (1999) proposed the function:

$$u(w;a) = \frac{e^{aw} - 1}{a}$$

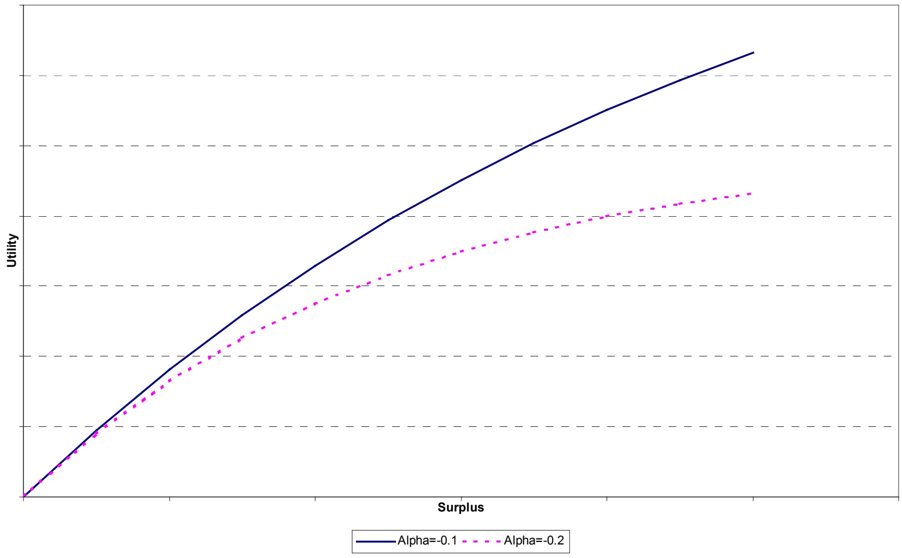
where:

u = utility
w = wealth
a = risk aversion parameter

Values of a <0 represent a risk averse preference. The function is plotted below for

a = -0.1and a = -0.2.

Figure 6.1: Exponential Utility Function



Diminishing marginal surplus is more pronounced as α becomes a larger negative value.

6.3 Parameterisation of the Utility Function

When satisfied with the form of our utility function, it is necessary to parameterise the function. This will require estimation of the insurer's risk tolerance.

Halliwell argues that it could be approached by allowing management to ponder the utility of several incremental wealth scenarios. He argues that the insurer's utility function could then be parameterised by selection of the parameter(s) which provide the best fit of the selected utility function to these subjective judgements.

Armed with an appropriate utility function, appropriately parameterised and a stochastic cash flow representing an insurance contract, we can then proceed to calculate an appropriate risk margin for the policy.

6.4 Systematic and Diversifiable Risk

It is instructive to consider the concepts of systematic and diversifiable risk in the context of utility theory. Utility theory recognises the concept of diversification as it impacts the distribution of surplus outcomes which occur as the insurer engages in a risky transaction. While this is not recognised explicitly, it is an outworking of the model, which considers the covariability of the profit outcome for an additional risk with the current stock of risks underwritten. The more positively correlated an additional risk with the existing book (all other things being equal), the greater the dispersion of the aggregate outcome, the smaller the level of the diversification and hence the greater the profit margin required by the insurer.

So, in a utility theory based approach to profit margin determination, diversifiable risk is rewarded to the extent that it is not eliminated by the relationship between the additional risk and the insurer's current stock of risks. This can be seen as a contrast with the Fairley model. The utility theory formulation is more in line with our comments in Section 2 regarding a model which rewards risk in line with the Feldblum formulation. However, utility theory goes further and suggests that it is the relationship with the present book of business, rather than a fully diversified insurance portfolio which drives the risk return tradeoff.

6.5 Some Intuitively Appealing Aspects of Utility Theory

As the utility function describes an insurer's attitude towards risk there is a certain intuitive appeal to the use of this approach for the calculation of risk margins. Is the pain associated with the loss of surplus ameliorated by the prospect of an increase in surplus associated with a favourable outcome? This statement of the solution framework seems well aligned with the problem we are trying to solve.

Another pleasing aspect of the application of utility theory is its ability to cope with pricing a risk / reward scenario where the outcome of the process is determined instantaneously. Under a discounted cash flow approach the risk loading is specified as a rate per time period, e.g. 17% per annum. A DCF approach cannot reasonably cope with pricing a risky scenario which is settled instantaneously. The appeal is more esoteric in the present context but nonetheless is a pleasing extension.

6.6 Criticisms

The major criticisms posed regarding utility theory centre on determination of the utility function.

Firstly, utility theory itself does not offer guidance on the functional form. As we have shown, exponential utility satisfies most of the major criteria but its tractability doesn't prove its appropriateness. If we cannot substantiate the correct functional form then it deeply impedes our ability to determine an appropriate profit margin.

Secondly, parameterising the function appears equally difficult. As noted, Halliwell discusses a methodology by which management would be asked to specify the utility derived under a range of surplus scenarios. Such an exercise would likely be fraught with practical difficulty.

Additionally with respect to exponential utility, the absence of "wealth effects" makes this particular form hard to justify. Halliwell suggests this can be overcome by choosing a value of a which varies inversely with the insurer's wealth. However, this suggestion would appear to make our exercise involving management discussed in the paragraph above inherently more difficult.

Conclusions

There are two major related considerations which must be borne in mind when contemplating an approach to determination of profit margins based on utility theory.

Firstly, the insurer centric view of risk and its price. Conceptually, the insurer demands a risk premium which is required to compensate it for engaging in a risky transaction such that its expected utility is not diminished. The implication is that there is reward for bearing not only parameter risk but process risk to the extent that it has not been eliminated by the insurer's current book of business.

Secondly the practical difficulty associated with the derivation of an insurer's utility function must be considered. The approach suggested by Halliwell seems problematic. It relies on a sufficient mathematical formulation of the views of the insurer's management on utility under a range of surplus scenarios. Because the formulation is by definition subjective in nature it is prone to a reasonable degree of estimation error and additionally may be subject to variations over time as sentiment changes. Additionally, the function so parameterised may introduce some unwanted bias in practice, if management and shareholder interests are not exactly aligned.

⑦ Proportional Hazards Transform

In this section, we introduce the Proportional Hazards (P-H) transform and discuss its application to insurance pricing. We explore P-H theory and define the P-H transform before highlighting several key features of the method and some of its potential limitations.

7.1 Background

The P-H method for pricing was first introduced by Wang in the mid-1990s and described as easy to understand, simple to use, and having some economic justification. Christofides (1998) presents some practical applications of the theory.

Proportional Hazards does not depend upon any financial or economic theory (e.g. CAPM). It simply associates a risk-loaded price to an index, ρ.

Broadly the method takes a random variable X and transforms its loss distribution in a particular way. The transformation is based on an index, ρ ($0 \le \rho \le 1$), known as the Risk Aversion Level ("RAL"). In practice this index depends upon the risk tolerance of the party pricing the risk in question. The expected value of the transformed loss distribution, the P-H mean, then determines the risk-adjusted premium at the chosen level of ρ.

7.2 The P-H Transform and Insurance

Wang (1995, 1998) develops P-H pricing theory based simply upon a non-negative loss random variable, X. Given a random variable X that describes a non-negative loss, the risk may be described by its decumulative distribution function (DDF) $S_X(x) = \Pr[X > x]$. The DDF is simply the probability that a loss exceeds some critical value, x. That is:

$$\begin{aligned} S_X(X) &= \Pr[X > x] \\ &= 1 - F(x) \end{aligned}$$

where F(x) is the cumulative distribution function.

The expected value of X, E[X], can be derived directly from $S_X(X)$, the DDF of X. It is the integral of its DDF over the range from zero to infinity, namely:

$$E[X] = \int_0^\infty S_X(x)dx$$

Two key elements of P-H theory depend on the DDF. These are:

- the P-H transform; and
- the P-H mean.

Given a best estimate loss distribution for X, $S_X(t) = \Pr[X > t]$ for some index ρ ($0 \le \rho \le 1$), the P-H transform is defined as follows:

$$S_Y(t) = [S_X(t)]^\rho$$

The P-H mean is the expected value of the transformed distribution. Given a best estimate loss distribution $S_X(t) = \Pr[X > t]$ for some index ρ ($0 \le \rho \le 1$) the P-H mean is defined as follows:

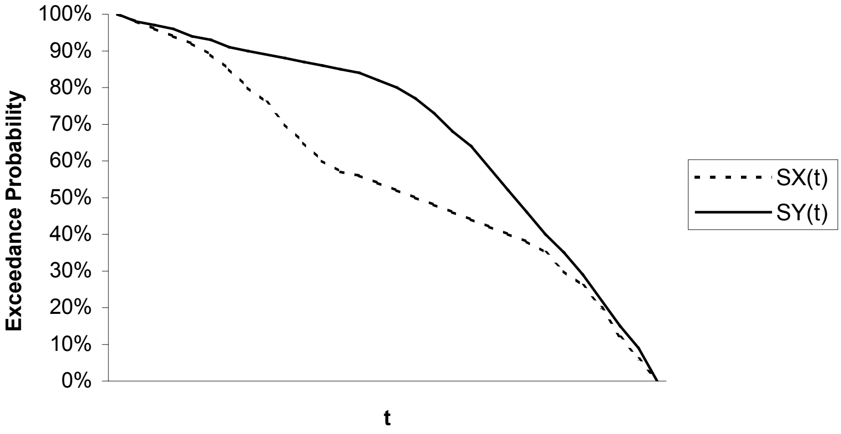
$$H_\rho(X) = \int_0^\infty [S_X(t)]^\rho\, dt \text{ for some } \rho\ (0 \le \rho \le 1)$$

The P-H mean was introduced by Wang to represent risk-adjusted premiums for insurance pricing. To calculate the risk-adjusted premium at any risk aversion level it is required to:

- determine the decumulative distribution function $S_X(t)$ of the underlying distribution;
- determine a risk aversion level, ρ; and
- integrate the P-H transform of this distribution, $S_X(t)^\rho$.

An arbitrary decumulative distribution function and its P-H transform are shown below in Figure 7.1. The areas under the curves give the mean (best estimate) risk premium and mean risk adjusted premium respectively. The area between the two curves is therefore the risk margin, the additional premium required to pay for uncertainty at the particular RAL, ρ.

Figure 7.1: A Decumulative Distribution Function and its P-H Transform



7.3 Some properties of the P-H risk adjusted premium

Wang shows that the P-H risk adjusted premium basis satisfies certain properties of a sound pricing approach. In general, for $0 \leq \rho \leq 1$, these properties include:

- Positive Loading: If the transformed decumulative distribution function S(y) is greater than the underlying S(x) then the expected value of Y will exceed the expected value of X.
- Linearity and scale invariance: $H_\rho(aX + b) = a\,H_\rho(X) + b$. This property avoids straightforward problems like those of scale conversion (e.g. a change of currency). It also allows the application of P-H to a quota share treaty. In this case $H_\rho(X) = H_\rho(aX) + H_\rho(\{1 - a\}X)$.
- Layer additivity: The total price of two adjacent excess of loss reinsurance layers should equal the price of one combined layer, at a given level of ρ. Thus for two adjacent excess of loss layers L(A,B) and L(B,C), under the P-H transform $H_\rho(L(A,C)) = H_\rho(L(A,B)) + H_\rho(L(B,C))$.

A further important property is that the P-H mean ensures an increasing relative loading for higher excess of loss layers for a given risk aversion level, ρ. This is consistent with market practice and demonstrates that P-H transforms can perform a useful role in pricing excess of loss reinsurance contracts.

Wang also illustrates a number of immediately practical features of the P-H transform.

For example, if X has a Pareto distribution with parameters (α,λ), then

$$S_X(t) \quad = \quad [\lambda \,/\, (\lambda + t)]^{\alpha}$$

In this case the P-H transform $S_Y(t)$ also has a Pareto distribution with parameters $(\rho\alpha,\lambda)$.

Not all loss distributions are so simple however. For example, when $S_X(t)$ has a gamma (or log-normal) distribution the P-H transform $S_Y(t)$ is no longer gamma (or log-normal). In fact, typically, loss distributions are intractable, often having a mass at zero and a long tail. Hence, it is likely that numerical integration will be necessary as algebraic methods are not possible.

7.4 Implications for Insurance Pricing

In practice, when pricing an insurance risk, the underlying loss distribution is rarely known with precision. Insufficient data or poor-quality data often results in sampling errors. Past data may not fully predict the future claim distribution due to changes in the claim generating mechanisms.

The P-H transform, S(Y), enables a risk adjusted premium to be calculated at any value of ρ once the decumulative distribution function has been obtained or estimated. This may be by algebraic or numerical integration.

The P-H transform, S(Y), may be considered as an upper confidence limit for the best-estimate loss distribution S(X), depending on RAL (see Wang, 1998). A smaller risk aversion level, ρ, results in a wider gap between the curves S(Y) and S(X) (figure 7.1). The RAL, ρ, may be set based on the degree of confidence there is in the estimated underlying loss distribution. The more uncertainty that is inherent to the situation, the lower the value of ρ that should be used.

The risk-adjusted premium will depend on the level of uncertainty and the risk aversion of the party taking the risk. It may assume a maximum value in a highly risk averse environment, perhaps equal to probable maximum loss. On the other hand the risk-adjusted premium may simply be set at its expected value, without any loading. In other words a risk-adjusted premium can be set at any level between its expected and maximum values.

In any event the P-H methodology provides a tool for making consistent risk adjustment with relative ease. It has direct application beyond simply calculating the profit loading for an individual contract thanks to certain properties that are important when pricing more complex excess of loss reinsurance contracts. The P-H transform lends itself well to deducing the value at risk by line of business in a capital allocation exercise.

7.5 Difficulties

The P-H transform's free constant, ρ, determines the level of the risk load. This constant is the risk aversion level.

Unfortunately, establishing ρ does not appear straightforward in practice. There is no universal ρ! Just as in Utility Theory, establishing ρ requires estimation of the risk taker's (ie insurer's) risk tolerance.

Indeed, within the small amount of literature that currently relates to P-H transforms, there is no mention of any specific way to derive the RAL. Instead practitioners typically deduce an implicit market RAL, based on market price.

Naturally, the value of the market implied RAL depends upon market conditions. The market price will be set somewhere between the expected value and a maximum value. The market RAL should therefore give an indication of market hardness or softness and help an insurer accept or decline a risk, given an estimate of its own RAL.

7.6 Final Remarks

The P-H transform method provides a concise methodology by which to adjust loss distributions and deduce a risk-adjusted price. As a risk adjustment tool the methodology appears to have a wide range of applications in insurance and elsewhere that pricing for uncertainty is required.

However, there is little published research regarding P-H theory and only a few practical examples are readily available. In practice it seems that establishing the risk aversion level relies ultimately upon judgment about an insurer's risk tolerance.

On balance, the relative newness of Proportional Hazards transforms leaves questions unanswered about its importance as a practical risk-pricing tool. There is certainly room for further research.

⑧ Conclusion

In this paper we have critically assessed a number of proposed methodologies for calculating a profit margin. We have discussed some of the fundamental issues and practical difficulties associated with each method.

The surplus-return method is intuitively appealing as it corresponds to the concept that the capital provided needs to earn an appropriate return. However, the allocation of capital to individual classes may be less intuitive, since the overall capital of the business is available to back the entire book, not just a part of it. Moreover, the method is dependent upon the methodology chosen to allocate capital and determine the required return. Both of these problems will result in the need to estimate parameters that can be difficult to determine in practice.

A CAPM based approach can be used to determine the required rate of return in the surplus-return method, or the discount rate for the Myers-Cohn methodology. The CAPM approach of rewarding risk by adding a loading to the risk-free rate is quite intuitive. However, the exact meaning of the liability beta is less clear and from a practical perspective the estimation of both the liability beta and, to a lesser extent, the market risk premium is fraught with difficulty. Additionally, the CAPM only rewards diversifiable risk. It is important to bear this in mind and consider its appropriateness in any practical application of the CAPM.

The Myers-Cohn approach also suffers from problems of parameter estimation. Key to the Myers-Cohn approach is the determination of the risk-adjusted discount rate, which is not straightforward. The discounted cash flow approach of the Myers-Cohn model is an intuitively appealing feature of this model, as is the aim of the Myers-Cohn formulation to be fair to both shareholders and policyholders. However, we would cite the difficulty with parameter estimation and the relative insensitivity to capital as definite shortcomings of this approach.

An option-pricing based approach does not rely on a parameter such as beta that is difficult to determine. The parameters of the option-pricing model are comparatively easier to determine, although estimation of the volatility parameter is still likely to be difficult. The option-pricing based approach is also appealing as both an option payment and an insurance payment is a payment contingent upon some event. However, although this model sounds reasonable, it suffers from two major problems. These are its reliance on the continuous hedging argument that is almost certainly inapplicable in insurance and the difficulty in finding an option type that adequately describes the insurance situation. These shortcomings mean that the validity of applying the option-pricing based approach in practice is questionable.

Both utility theory and proportional hazard transforms are subject to the problem of parameter estimation, depending upon a risk aversion parameter for which no obvious value exists.

Utility theory has an additional problem of determining an appropriate utility function, which is a non-trivial exercise. Intuitively, utility theory is appealing as it represents a close alignment of the mathematical solution with a series of key features that can be seen as integral to the insurance situation. The nature of the risk rewarded by utility theory takes into account the current situation of the insurance company, which is also appealing. However, one of the most widely cited solutions for the utility function, the exponential function, results in a profit loading that is independent of the level of wealth which is contrary to widely held views.

In contrast to utility theory, proportional hazards transforms uses loss distribution functions as a starting point, which may be better known although are difficult to estimate in practice. The method depends upon a risk-aversion parameter that is also difficult to estimate, although it may be possible to obtain a reasonable starting point by inferring the risk-aversion level from market information. One key strength of the proportional hazards methodology is the consistent treatment of risk across different categories of business.

No one methodology dominates the others in terms of theoretical strength, intuitive strength and practicality. Each of the methodologies considered has some weakness or inherent difficulty. Ultimately, the choice of method is likely to be largely driven by practical considerations regarding the constraints of the current situation and the availability of suitable inputs. However, it is important to bear in mind the strengths and weaknesses of the methodologies considered.

In practice the insurance business must compete for the capital required to run the business, both with other insurance businesses and with other industries. In order to attract the required capital the business must offer a competitive return. The insurance company management is charged with the responsibility of delivering this required return to shareholders.

To meet this objective, management will generally target a particular required rate based on the market conditions. Whilst ultimately, the pricing actuary must deliver a set of rates which attempt to meet this objective he or she should be prepared to assess the appropriateness of this objective critically. In this paper we have discussed a range of approaches which may be used by the pricing actuary to challenge and test this required return.

Part II

Ⓐ Bibliography

Christofides, S (1998), "Pricing for Risk in Financial Transactions", Institute of Actuaries General Insurance Convention, Glasgow.

Cummins, J.D. (1990) "Asset Pricing Models and Insurance Ratemaking", ASTIN Bulletin, Vol 20:2, pp 125-166, International Actuarial Association, Brussels.

D'Arcy, Dyer (1997), "Ratemaking: A Financial Economics Approach", Proceedings of the Casualty Actuarial Society LXXXIV, Casualty Actuarial Society, Arlington.

D'Arcy, S.P., Garven, J.R. (1990), "Property-Liability Insurance Pricing Models: An Empirical Evaluation", Journal of Risk and Insurance, Vol 57, No 2, pp 391 – 430.

Fairley, William (1979), "Investment Income and Profit Margins in Property-Liability Insurance: Theory and Empirical Results" in Fair Rate of Return in Property-Liability Insurance, Cummins J.D., Harrington, S.A. (Eds), pp 1-26, Kluwer Nijhoff Publishing, Boston, 1987.

Feldblum, Shalom (1990), "Risk Loads for Insurers", Proceedings of the Casualty Actuarial Society LXXVII, pp 160-195, Casualty Actuarial Society, Arlington.

Gogol, Daniel, F (1996), "Pricing to Optimize an Insurer's Risk-Return Relation", Proceedings of the Casualty Actuarial Society, 83, pp 41-74, Casualty Actuarial Society, Arlington.

Halliwell (1999), "ROE, Utility and the Pricing of Risk", Casualty Actuarial Society Forum, Vol: Spring, pp 71-136, Casualty Actuarial Society, Arlington.

Heckman, Philip E. (1992), "Some Unifying Remarks on Risk Load", Casualty Actuarial Society Forum, Vol: Spring, pp 31-44, Casualty Actuarial Society, Arlington.

Hart, Buchannan, Howe (1996), Actuarial Practice in General Insurance, Institute of Actuaries of Australia, Sydney, Australia.

Kozik, Thomas (1994), “Underwriting Betas – The Shadows of Ghosts”, Proceedings of the Casualty Actuarial Society LXXXI, pp 303-329, Casualty Actuarial Society, Arlington.

Kreps, Rodney (1998), “Investment-Equivalent Reinsurance Pricing” Proceedings of the Casualty Actuarial Society, 85, pp 101-131, Casualty Actuarial Society, Arlington.

Kreps, Rodney (1990), “Reinsurer Risk Loads from Marginal Surplus Requirements”, Proceedings of the Casualty Actuarial Society, LXXVII, pp 196 – 203, Casualty Actuarial Society, Arlington.

Mahler (1998), “The Myers-Cohn Profit Model, A Practical Application” Proceedings of the Casualty Actuarial Society, 85, Casualty Actuarial Society, Arlington.

Meyers, Glenn (1996), “The Competitive Market Equilibrium Risk Load Formula for Catastrophe Ratemaking”, Proceedings of the Casualty Actuarial Society, LXXXIII, pp 563-600, Casualty Actuarial Society, Arlington.

Myers, Stewart and Cohn, Richard (1981), “A Discounted Cash Flow Approach to Property-Liability Rate Regulation” in Fair Rate of Return in Property-Liability Insurance, Cummins J.D., Harrington, S.A. (Eds), pp 1-26, Kluwer Nijhoff Publishing, Boston, 1987.

Philbrick, Stephen (1991), “A Discussion of Feldblum’s 1990 paper ‘Risk Loads for Insurers’”, Proceedings of the Casualty Actuarial Society LXXVIII, Casualty Actuarial Society, Arlington.

Venter, G (1998), “A Discussion: Implementation of Proportional Hazards Transforms in Ratemaking”, Proceedings of the Casualty Actuarial Society LXXXV, pp 980-990, Casualty Actuarial Society, Arlington.

Wang, Shaun (1995), “Insurance Pricing and increased limits ratemaking by proportional hazards transforms”. Insurance Mathematics and Economics, 17, pp 43-54.

Wang, Shaun (1998), “Implementation of Proportional Hazards Transforms in Ratemaking”. Proceedings of the Casualty Actuarial Society, LXXXV, Casualty Actuarial Society, Arlington.